RAN-2203000205023005

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T.Y.B.Sc. Mathematics (Sem. - V) Examination October - 2025

MTH - 505 - Graph Theory (old)

[Total Marks: 50

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
(2) All questions are compulsory
(3) Follow usual notations
(4) Figures to the right indicate marks of the question
(5) Total marks 50.

Seat No.:

Student's Signature

Q. 1 Answer the following (Any FIVE) **[10]**

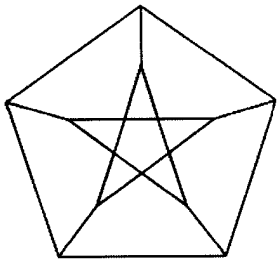
(1) Construct a complete graph with four and five vertices.
(2) If G is a simple graph with 15 vertices, then find the maximum number of edges in G, also write the maximum possible degree of a vertex in G .
(3) Draw the graphs of the chemical compounds: N_2O_3 and C_6H_6.
(4) Draw a graph that has Hamiltonian path but not a Hamiltonian circuit.
(5) Prove that every self loop is a circuit but converse is not true.
(6) What is the maximum number of edges in a simple graph with 12 vertices and 5 components?
(7) Find the number of edges in a connected graph without any circuit contains 8 vertices.
(8) Find the number of pendant vertex in a binary tree with 2025 vertices.

Q. 2 Answer the following (Any TWO) **[10]**

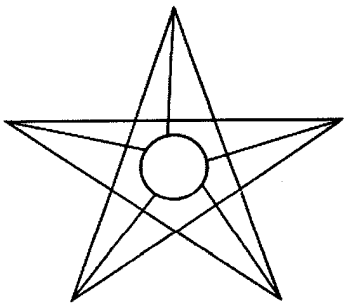
(1) If $v_1, v_2,...,v_n$ are n vertices and e is the total number of the edges in a graph G, then prove that $\sum_{i=1}^{n} d(v_i) = 2e$.
(2) State and solve Utility problem of three houses and three utilities namely, water, gas and electricity.
(3) Check the validity of the following statements. Correct it if it is invalid.
   1. An infinite graph with finite number of edges must have finite number of Isolated vertices.
   2. An infinite graph with finite number of vertices must have at least one pair of vertices joined by finite number of edges.

Q. 3 Answer the following (Any TWO) **[10]**

(1) State necessary conditions for two graphs to be isomorphic. Are the following graphs G_1 and G_2 isomorphic? Justify your answer.



Graph G_1



Graph G_2

(2) Prove that a connected graph G remains connected after removing an edge e_i from the graph G, if and only if e_i is in some circuit in G.

(3) Show that a graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty disjoint subsets V_1 and V_2 in which there is no edge in G whose one end vertex is in V_1 and the other in V_2.

Q. 4 Answer the following (Any TWO) **[10]**

(1) Prove that in a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges.

(2) Draw a graph in which an Euler graph is also Hamiltonian circuits. What Can you say about such graph in general?

(3) Prove that a graph containing m edges namely, $e_1, e_2, ..., e_m$, can be decomposed in $(2^{m-1} - 1)$ different ways into pairs of subgraphs g_1 and g_2.

Q. 5 Answer the following (Any TWO) **[10]**

(1) Prove that every tree has either one or two centers.

(2) Prove that a graph is a tree if and only if it is minimally connected.

(3) If the total number of vertices in a Binary tree is n, then prove that the number of pendant vertices in it is $\left(\frac{n+1}{2}\right)$

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